4.1 Further Differentiation

Question Paper

Course	CIE A Level Maths
Section	4. Differentiation
Topic	4.1 Further Differentiation
Difficulty	Very Hard

Time allowed: 60

Score: /45

Percentage: /100

Question 1

Use an appropriate method to differentiate each of the following.

(i) $\tan 3x + e^{7-2x^2}$

(ii) $(x^2 + 2x - 8)\cos(3 - x)$

(iii) $\dfrac{\ln 7x}{\sin(x^2+5)}$

(iv) $\sqrt{\cos 4x}$

[8 marks]

Question 2

A curve has the equation $y = 3^x + 2^{-x}$.

Show that the gradient of the normal to the curve at the point $\left(1, \frac{7}{2}\right)$ is

$$\frac{2}{\ln 2 - 6 \ln 3}$$

[4 marks]

Question 3

Find the derivative of the function $f(x) = \sin\left(\cos\left(\ln\frac{1}{x}\right)\right),\ x > 0$.

[4 marks]

Question 4a

(a) Show that the derivative $y = 4^{-x^4}$ is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -(\ln 4)x^3 4^{1-x^4}$$

[4 marks]

Question 4b

(b) Hence find the equation of the tangent to the curve at the point $\left(1, \frac{1}{4}\right)$, giving your answer in the form $y = ax + b$, where a and b are to be given as exact values.

[2 marks]

Question 5a

Differentiate with respect to x, simplifying your answers where possible:

(a) $(5 + \sin^2 3x)e^{x^2-3x+2}$

[3 marks]

Question 5b

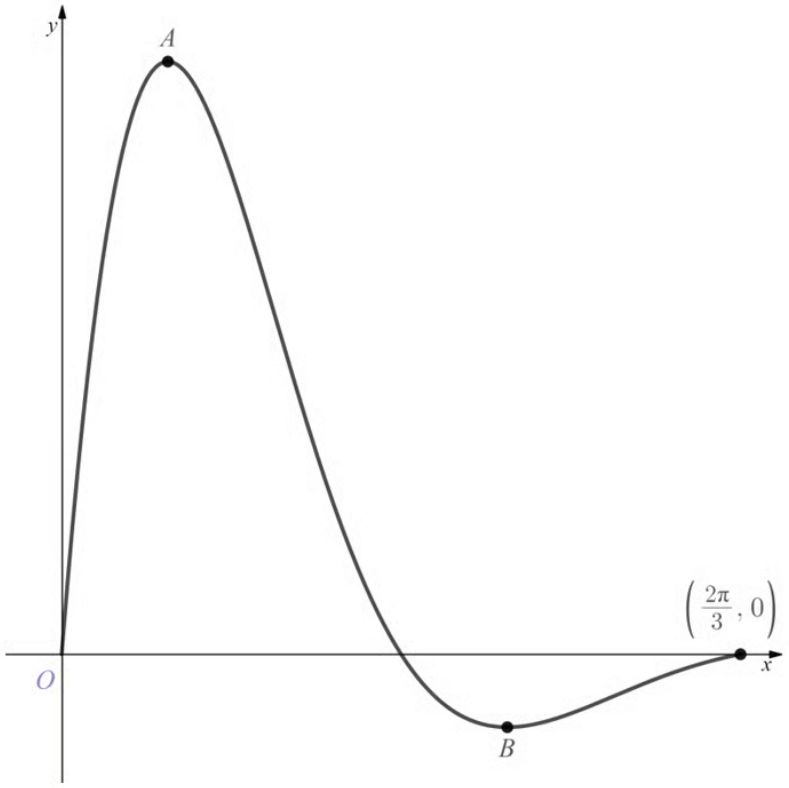
(b) $3^{\sqrt{x}}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$

[3 marks]

Question 6

The diagram below shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = \frac{\sin 3x}{e^{2x-3}}, \qquad 0 \leq x \leq \frac{2\pi}{3}$$



The points A and B are maximum and minimum points, respectively.

Find the range of $\mathrm{f}(x)$, giving your answer correct to 3 decimal places.

[6 marks]

Question 7

A sequence of functions is defined by the recurrence relation

$$u_{k+1}(x) = \frac{\mathrm{d}}{\mathrm{d}x}u_k(x), \;\; u_1(x) = \sin\left(x\sqrt{2}\right)$$

Based on that sequence, the function $\mathrm{f}_n(x)$ is defined by

$$\mathrm{f}_n(x) = \sum_{r=1}^{n} u_r(x)$$

Calculate the value of $\mathrm{f}_{41}\left(\frac{\pi\sqrt{2}}{4}\right)$

[5 marks]

Question 8

Use calculus to find the coordinates of the stationary points of the curve

$$y = \frac{x}{3} - \tan^{-1}\left(\frac{2x}{3}\right)$$

and determine whether each one is a maximum or a minimum. The coordinates should be given as exact values.

[6 marks]